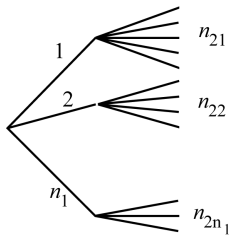
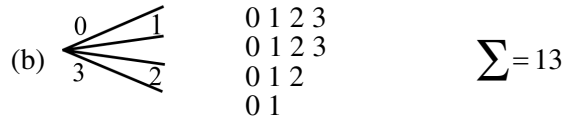


# Chapter 1

1.1



(a) 
$$\sum_{i=1}^{n_1} n_{2i}$$

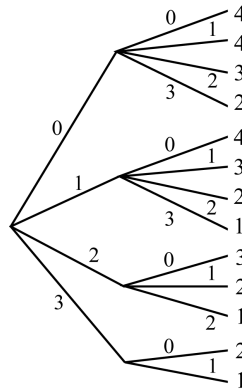


1.2 
$$\sum_{i=1}^{n_1} n_2 i = \sum_{i=1}^{n_1} n_2 = n_1 n_2$$

1.3

(a)

$n_{300} = 4$	$n_{320} = 3$
$n_{301} = 4$	$n_{321} = 2$
$n_{302} = 3$	$n_{322} = 1$
$n_{303} = 2$	$n_{330} = 2$
$n_{310} = 4$	$n_{331} = 1$
$n_{311} = 3$	
$n_{312} = 2$	
$n_{313} = 1$	



(b) 
$$\Sigma = 4 + 4 + 3 + \dots + 2 + 1 = 32$$

1.4 
$$\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} n_2 = n_1 n_2 n_3$$

1.5 (b) 6, 20, and 70

$$\text{"2 out of 3"} \quad m = 2 \quad 2 \left[ \binom{1}{1} + \binom{2}{1} \right] = 2(1+2) = 6$$

$$\text{"3 out of 5"} \quad m = 3 \quad 2 \left[ \binom{2}{2} + \binom{3}{2} + \binom{4}{2} \right] = 2(1+3+6) = 20$$

$$\text{"4 out of 7"} \quad m = 4 \quad 2 \left[ \binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \binom{6}{3} \right] = 2(1+4+10+20) = 70$$

1.6 (a)  $10! \approx \sqrt{20\pi} \left( \frac{10}{e} \right)^{10} = (7.92665)(3.678797)^{10} = (7.92665)(454,002.49) = 3,598,719$

$$\% \text{ error} = \frac{3.6288 - 3.5987}{3.6288} \cdot 100 = 0.83\%$$

$$12! \approx \sqrt{24\pi} \left( \frac{12}{e} \right)^{12} = (8.683215)(4.41455)^{12} = 475,683,224$$

$$\% \text{ error} = \frac{4.7800 - 4.7568}{4.7900} \cdot 100 = 0.69\%$$

$$\begin{aligned} \text{(b)} \quad \binom{52}{13} &= \frac{52!}{13! 39!} = \frac{\sqrt{104\pi} \left( \frac{52}{e} \right)^{52}}{\sqrt{26\pi} \sqrt{78\pi} \left( \frac{13}{e} \right)^{13} \left( \frac{39}{e} \right)^{39}} \\ &= \frac{13^{52} \cdot 4^{52}}{\sqrt{19.5\pi} 13^{13} \cdot 13^{39} \cdot 3^{39}} = \frac{4^{52}}{\sqrt{19.5\pi} 3^{39}} = 639 \text{ billion} \end{aligned}$$

1.7 Using Stirling's formula in  $\binom{2n}{n} = \frac{2n!}{n! n!}$  yields

$$\frac{\binom{2n}{n} \sqrt{\pi n}}{2^{2n}} = \frac{\sqrt{4\pi n} \left( \frac{2n}{e} \right)^{2n}}{\left[ \sqrt{2\pi n} \left( \frac{\pi}{e} \right)^n \right]^2} \cdot \frac{\sqrt{\pi n}}{2^{2n}} = 1$$

1.8  $n^r$  and  $12^3 = 1,728$

$$1.9 \quad \binom{r+n-1}{r} \text{ and } \binom{5+3-1}{5} = \binom{7}{5} = 21$$

1.10 Substitute  $r-n$  for  $r$  into result of 1.9

$$\binom{r-n+n-1}{r-n} = \binom{r-1}{r-n} \text{ and } \binom{5-1}{5-3} = \binom{4}{2} = 6$$

1.11 (b) Seventh row is 1, 6, 15, 20, 15, 6, 1

Eighth row is 1, 7, 21, 35, 35, 21, 7, 1

$$(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^2y^3 + 15x^2y^4 + 6xy^5 + y^6$$

$$(x + y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

- 1.14 (a) Set  $x = 1$  and  $y = 1$   
 (b) Set  $x = 1$  and  $y = -1$   
 (c) Set  $x = 1$  and  $y = a - 1$

1.19 (a) 
$$\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{24} = -\frac{15}{384} \text{ and } \frac{(-3)(-4)(-5)}{6} = -10$$

(b) 
$$\begin{aligned} \sqrt{5} &= 2\left(1 + \frac{1}{4}\right)^{\frac{1}{2}} = 2\left[1 + \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{1}{4}\right)^2 + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^3\right] \\ &= 2\left[1 + \frac{1}{8} - \frac{1}{64} + \frac{3}{512}\dots\right] = 2 \cdot \frac{512 + 64 - 8 + 3}{512} \\ &= 2 \cdot \frac{571}{512} = 2.23 \end{aligned}$$

$$\frac{1142}{512} = 2.230$$

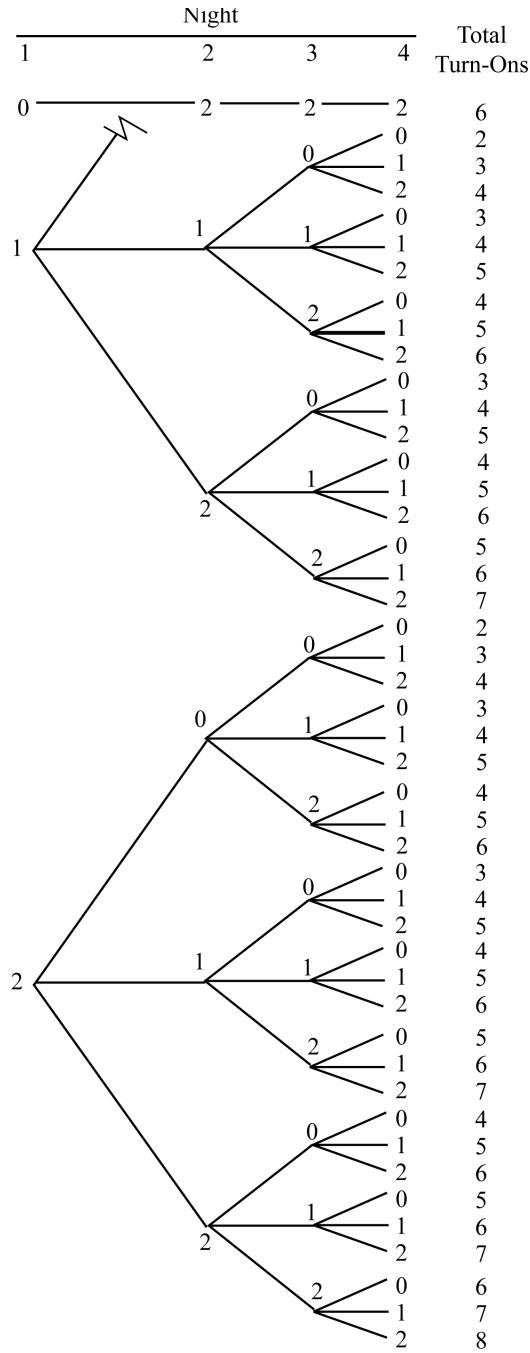
1.20 (a) 
$$\frac{(-1)(-2)\dots(-r)}{r!} = (-1)^r$$

(b) 
$$\begin{aligned} \binom{-n}{r} &= \frac{(-n)(-n-1)\dots(-n-r+1)}{r!} = (-1)^r \frac{n(n+1)\dots(n+r-1)}{r!} \\ &= (-1)^r \frac{(n+r-1)\dots(n+1)n}{r!} = (-1)^r \binom{n+r-1}{r} \end{aligned}$$

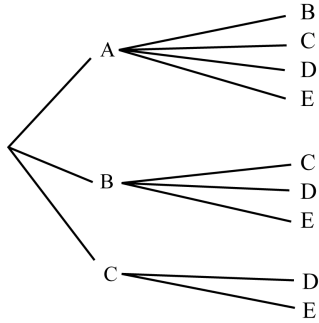
1.21 
$$\frac{8!}{2! 3! 3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{2 \cdot 6} = 560$$

1.22 
$$\frac{9!}{3! 2! 3!} \cdot 2^3 \cdot 3^2 \cdot (-4)^3 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{12} \cdot 8 \cdot 9 \cdot 64 = -23,224,320$$

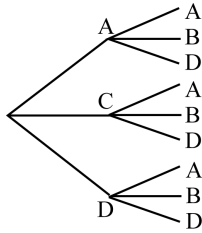
1.24 Note: If there are 0 turn-ons the first night, 6 turn-ons in four nights can only occur if there are 2 turn-ons on each of the subsequent three nights. Thus, we need to show only that part of the tree following this event.



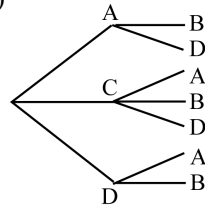
1.25



1.26 (a)



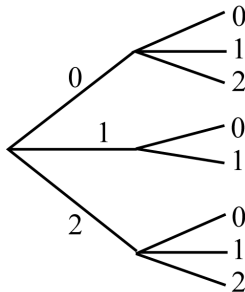
(b)



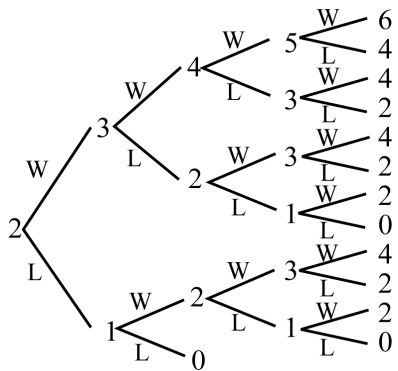
1.27 (a) 5

(b) 4

1.28



1.29



1.30 (a)  $6 \cdot 5 = 30$ ; (b)  $6 \cdot 6 = 36$ ;

1.31 (a) 6; (b)  $6 \cdot 5 = 30$ ; (c)  $5 \cdot 4 = 20$  first one fixed; (d)  $6 + 30 + 20 = 56$

1.32 (a)  $4 \cdot 5 \cdot 2 = 40$ ; (b)  $5 \cdot 6 \cdot 3 = 90$

1.33 (a)  $5 \cdot 4 = 20$ ; (b)  $5 \cdot 4 \cdot 3 = 60$

1.34  $3^{15} = 14,348,907$

1.35  $\frac{15 \cdot 14}{2 \cdot 1} = 105$

1.36 (a)  $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ ; (b)  $\frac{5040}{24} = 210$

1.37 (a)  $\frac{14 \cdot 13}{2 \cdot 1} = 91$ ; (b)  $\frac{14 \cdot 13 \cdot 12}{3 \cdot 2 \cdot 1} = 364$

1.38  $6! = 720$

1.39  $\frac{6!}{2! 2! 2!} = \frac{720}{8} = 90$

1.40  $5! = 120$  and  $120 - 2 \cdot 4! = 72$

1.41  $7! = 5040$

1.42 (a)  $5! = 120$ ; (b)  $\frac{5!}{2!} = 60$

1.43  $\frac{10!}{3! 3! 2!} = \frac{3628800}{72} = 50,400$  and  $\frac{8!}{3! 2!} = \frac{40320}{12} = 3360$

1.44  $\frac{10!}{5! 4!} = \frac{3628800}{120 \cdot 24} = 1,260$

1.45  $\frac{8!}{3! 4!} = \frac{40320}{6 \cdot 24} = 280$

1.46 (a)  $\binom{20}{7} = 77,520$ ; (b)  $\binom{20}{10} = 184,755$

(c)  $\binom{20}{17} + \binom{20}{18} + \binom{20}{19} + \binom{20}{20} = 1140 + 190 + 20 + 1 = 1351$

1.47 (a)  $\binom{7}{2} = 21$ ; (b)  $\binom{4}{2} = 6$ ; (c)  $3 \cdot 4 = 12$

$$1.48 \quad \binom{3}{2}\binom{7}{2} + \binom{3}{3}\binom{7}{1} = 3 \cdot 21 + 1 \cdot 7 = 63 + 7 = 70$$

$$1.49 \quad \binom{4}{2}\binom{7}{3}\binom{3}{1} = 6 \cdot 35 \cdot 3 = 630$$

$$1.50 \quad \binom{13}{5}\binom{13}{3}\binom{13}{3}\binom{13}{2} = 1287 \cdot 286 \cdot 286 \cdot 78 = 8,211,173,256$$

$$1.51 \quad \frac{7!}{3!2!} = \frac{5040}{12} = 420$$

$$1.52 \quad 3^{10} = 59,049$$

$$1.53 \quad 5^5 = 15,625$$

$$1.54 \quad \binom{12+6-1}{12} = \binom{17}{12} = \binom{17}{5} = 6,188$$

$$1.55 \quad \binom{12-1}{6} = \binom{11}{6} = 462$$

$$1.56 \quad \binom{14+3-1}{14} = \binom{16}{14} = 120$$

$$1.57 \quad \binom{r-2n+n-1}{n-1} = \binom{r-n-1}{n-1}$$

$$\binom{r-n-1}{n-1} = \binom{10}{2} = 45$$